The correlation coefficient

The correlation coefficient – often designated by 'r' – is a popular coefficient in statistical analysis. Its value is confined in the interval [1, 1]. In this particular simulation the coefficient became -0.862. The minus sign indicates that the slope of the model is negative (but not more than that).
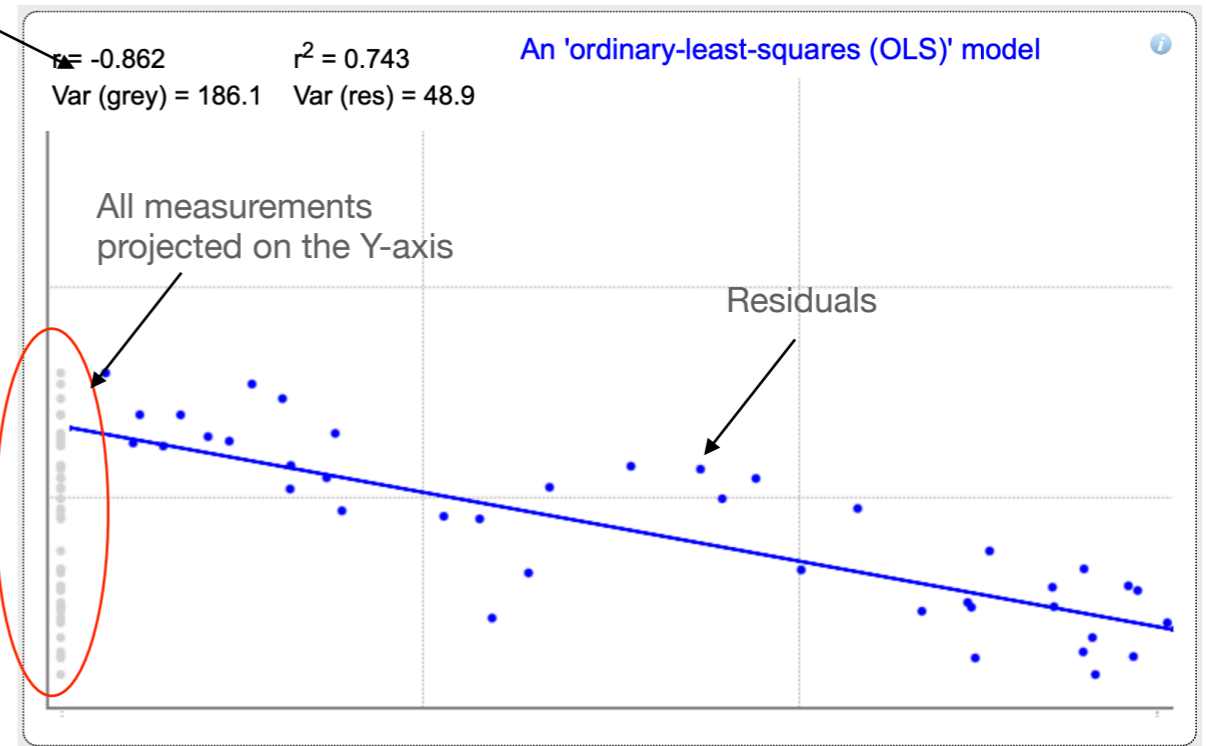
r = -0.862 r^2 = 0.743
Var (grey) = 186.1   Var (res) = 48.9

The r-squared shows how large a proportion of the total variance ("Var (grey)") that is 'explained' by the model.

The rest is the variance of the residuals (blue dots).

The residuals contains the variation from sources of variation not included in the analysis.

A rerun of the simulation, with the slide for 'Residual sigma' set to 0, shows a coefficient as 1:

r = -1.000 r^2 = 1.000
Var (grey) = 73.7   Var (res) = 0.0

r = -0.862 r^2 = 0.743
Var (grey) = 186.1   Var (res) = 48.9

An 'ordinary-least-squares (OLS)' model

All measurements projected on the Y-axis

Residuals

$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

A common expression of the correlation coefficient