Notation	$\xi = \frac{x-\mu}{\sigma},\ \alpha = \frac{a-\mu}{\sigma},\ \beta = \frac{b-\mu}{\sigma}$ $Z = \Phi(\beta) - \Phi(\alpha)$
Parameters	$\mu \in \mathbb{R}$ $\sigma^2 \geq 0$ (but see definition) $a \in \mathbb{R}$ — minimum value of x $b \in \mathbb{R}$ — maximum value of x ($b > a$)
Support	$x \in [a, b]$
PDF	$f(x; \mu, \sigma, a, b) = \frac{\varphi(\xi)}{\sigma Z}$ [1]
CDF	$F(x; \mu, \sigma, a, b) = \frac{\Phi(\xi) - \Phi(\alpha)}{Z}$
Mean	$\mu + \frac{\varphi(\alpha) - \varphi(\beta)}{Z}\sigma$
Median	$\mu + \Phi^{-1}\left(\frac{\Phi(\alpha) + \Phi(\beta)}{2}\right)\sigma$
Mode	$\begin{cases} a, & \text{if } \mu < a \\ \mu, & \text{if } a \leq \mu \leq b \\ b, & \text{if } \mu > b \end{cases}$
Variance	$\sigma^2\left[1 - \frac{\beta\varphi(\beta) - \alpha\varphi(\alpha)}{Z} - \left(\frac{\varphi(\alpha) - \varphi(\beta)}{Z}\right)^2\right]$
Entropy	$\ln(\sqrt{2\pi e}\sigma Z) + \frac{\alpha\varphi(\alpha) - \beta\varphi(\beta)}{2Z}$
MGF	$e^{\mu t + \sigma^2 t^2/2}\left[\frac{\Phi(\beta - \sigma t) - \Phi(\alpha - \sigma t)}{\Phi(\beta) - \Phi(\alpha)}\right]$

Two sided truncation:

Let $\alpha = (a - \mu)/\sigma$ and $\beta = (b - \mu)/\sigma$. Then:

$$\mathrm{E}(X \mid a < X < b) = \mu - \sigma\frac{\varphi(\beta) - \varphi(\alpha)}{\Phi(\beta) - \Phi(\alpha)}$$

and

$$\mathrm{Var}(X \mid a < X < b) = \sigma^2\left[1 - \frac{\beta\varphi(\beta) - \alpha\varphi(\alpha)}{\Phi(\beta) - \Phi(\alpha)} - \left(\frac{\varphi(\beta) - \varphi(\alpha)}{\Phi(\beta) - \Phi(\alpha)}\right)^2\right]$$

One sided truncation (of lower tail):

In this case $b = \infty,\ \varphi(\beta) = 0,\ \Phi(\beta) = 1,$ then

$$\mathrm{E}(X \mid X > a) = \mu + \sigma\varphi(\alpha)/Z,$$

and

$$\mathrm{Var}(X \mid X > a) = \sigma^2[1 + \alpha\varphi(\alpha)/Z - (\varphi(\alpha)/Z)^2],$$

where $Z = 1 - \Phi(\alpha)$.

One sided truncation (of upper tail):

In this case $a = \alpha = -\infty,\ \varphi(\alpha) = 0,\ \Phi(\alpha) = 0,$ then

$$\mathrm{E}(X \mid X < b) = \mu - \sigma\frac{\varphi(\beta)}{\Phi(\beta)},$$

$$\mathrm{Var}(X \mid X < b) = \sigma^2\left[1 - \beta\frac{\varphi(\beta)}{\Phi(\beta)} - \left(\frac{\varphi(\beta)}{\Phi(\beta)}\right)^2\right].$$

Some R-commands

```
mu    <- 50
sigma <- 5
n     <- 10000

lowlimit <- 47
hilimit  <- 55

res1 <- rnorm(n, mu, sigma)

res1a <- res1[res1 > lowlimit]
res1b <- res1[res1 > lowlimit & res1 < hilimit]

hist(res1)
hist(res1a)
hist(res1b)

res2a <- ifelse(res1 < lowlimit, lowlimit, res1)
hist(res2a)
```