

Two normal distributions and five requirements

Frequently the very mathematics or statistics of a problem is fairly straightforward. Instead, the formulation of the problem might need some extra thinking and this document discusses such a problem – a real case slightly disguised.

Suppose that we have a process that welds two seams of a plastic bag containing a liquid. The liquid is released (for a medical purpose not discussed here) by squeezing the bag by hand until the weaker seam breaks. There are five requirements – the seams must not be too weak or too strong and for each bag the weaker seam must be the one that breaks.

If we designate the weak seam 'Low force' and the strong seam 'High force' we can formulate the five requirements:

1. Lower limit of 'Low force'
2. Upper limit of 'Low force'
3. Lower limit of 'High force'
4. Upper limit of 'High force'
5. High force > c*Low force

The fifth requirement states that the strong seam must be a certain constant ('c') stronger than the weak seam. We assume that the two variables (the strength of the two seams) are independent of one another.

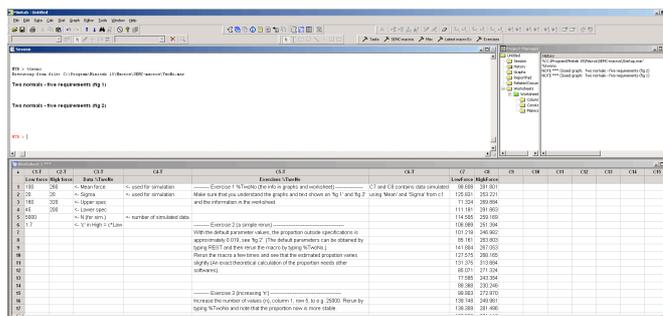
It turns out that the problem must be formulated as a bivariate normal distribution and that the 'approved result' must be a square restricted by the upper and lower limits and a straight line (the 5th requirement), see the diagram 'Two normal... (fig 2)' below below.

The Minitab macro %TwoNo (available by request) performs the simulation using parameters by default or set by the user.

1. Introduction of the macro (%TwoNo)
2. Two normal distributions – five requirements (fig 1)
3. Two normal distributions – five requirements (fig 2)
4. Summary

1. Introduction of the macro (%TwoNo)

The macro is started by typing %TwoNo at the Minitab-prompt. At the initial run, the macro uses the parameters stored in column c1 and c2 but different values can be used at a rerun. See also column c5 for suggested exercises. All the simulated data values are stored in the worksheet making it possible to use the standard menus on the data. (The figure below is a screen shot of the macro after an initial run, the created diagrams are hidden).

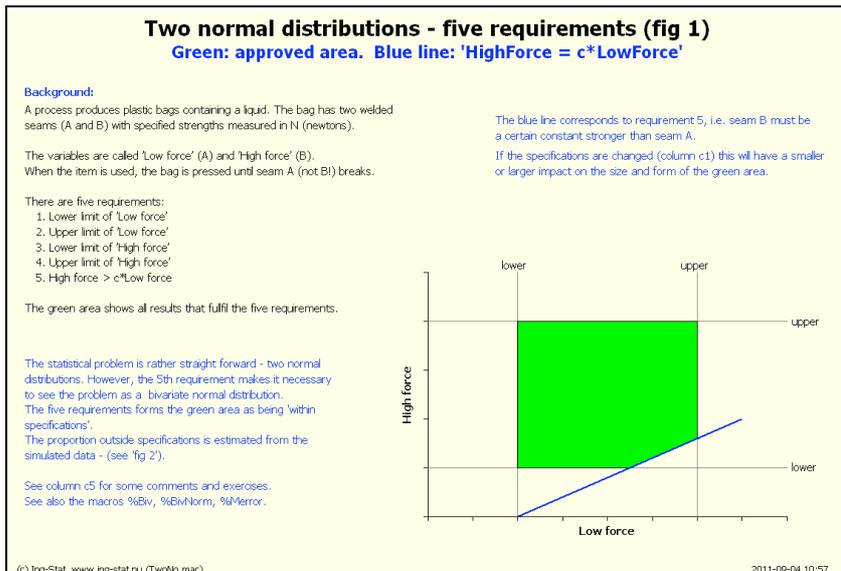


A screen shot of the screen after an initial run of the macro %TwoNo.

2. Two normal distributions – five requirements (fig 1)

The diagram below, created by the macro, describes briefly the problem and lists the five requirements. It also shows the approved outcomes as a green area restricted by the upper and lower limits of each of the two variables. The blue line corresponds to the fifth requirement.

The fifth requirement is the usual expression for a straight line without an intercept term. The constant 'c' is usually called the slope of the line. It is of course not necessary that the fifth requirement restricts the area as in the diagram below. If the upper limit of 'Low force' and or the lower limit of 'High force' or the constant 'c' are changed, the green area might get a rectangular shape. If the process fulfills the requirement then it is of course highly unlikely getting an outcome in the extreme corners of the green area. The diagram 'Two normal... (fig 2)' illustrates this.

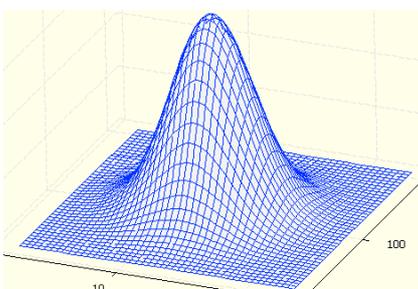


This display describes the problem and lists the five requirements.

The green area corresponds to approved outcomes of the variables.

3. Two normal distributions – five requirements (fig 2)

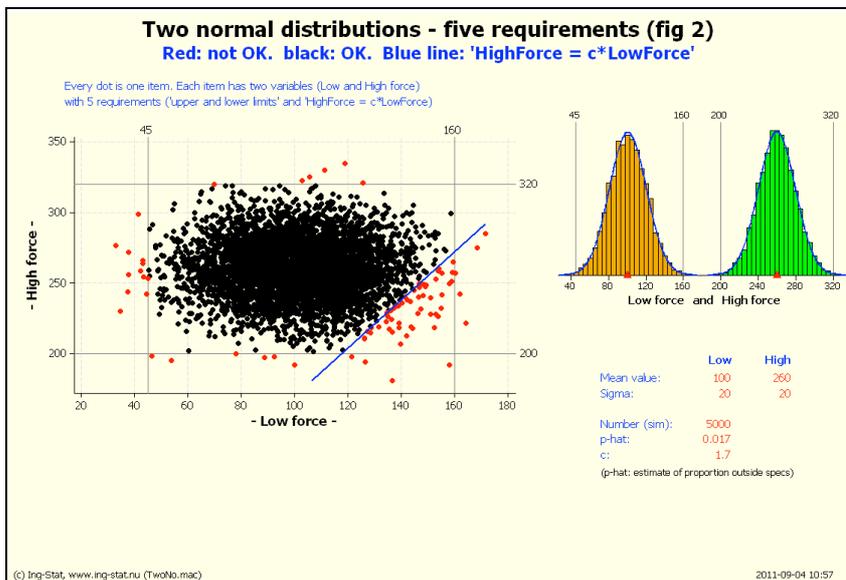
The graph 'fig 2' on next page contains two histograms and corresponding normal distributions. This is of course one of the more common ways to graphically show the result. However, as the problem can be described as a bivariate normal it is necessary to show the extra dimension. (In theoretical discussions a bivariate normal distribution is often shown as a 3D-diagram, see e.g. the macros %BivNorm for an illustration).



A screen shot of a theoretical presentation of a bivariate normal distribution. (Created by the macro %BivNorm.)

The left diagram below is an ordinary scatter diagram where every dot is an item (here plastic bag), plotted with its measured results of the strong and weak seam. The red dots represent the result outside the requirements. The proportion outside the limits is estimated from the simulation and

written on the diagram. (A theoretical calculation of the this proportion requires some other software.) By rerunning the macro different parameter setting can be used, including the number of data points simulated and constant 'c', the fifth requirement.



The graph to the left contains two histograms, one for 'Low force' and one for 'High force'. The scatter diagram shows the simulated outcome of the two variables.

4. Summary

This document discusses a rather simple situation involving two normal distributions but where an extra requirement makes the formulation slightly more difficult. By considering the problem as a bivariate situation it becomes simpler. We do not involve the theoretical bivariate distribution, as it would disturb the discussion.

Of course, the bivariate – and the natural extension – the multinomial normal distribution are of course worth knowing but it needs a more increased mathematical handling to be mastered. ■